OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4733/01

MATHEMATICS

Probability & Statistics 2

QUESTION PAPER

TUESDAY 10 JUNE 2014: Morning

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4733/01
List of Formulae (MF1)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

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INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

If you use the Printed Answer Book, write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.

The total number of marks for this paper is 72.

Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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1 The random variable F has the distribution B(50, 0.7). Use a suitable approximation to find P($F > 40$). [5]

2 The events organiser of a school sends out invitations to 150 people to attend its prize day. From past experience the organiser knows that the number of those who will come to the prize day can be modelled by the distribution B(150, 0.98).

(i) Explain why this distribution cannot be well approximated by either a normal or a Poisson distribution. [3]

(ii) By considering the number of those who do not attend, use a suitable approximation to find the probability that fewer than 146 people attend. [4]

3 The random variable G has the distribution $\mathrm{N}(\mu, \sigma^2)$. One hundred observations of G are taken. The results are summarised in the following table.

Interval	$G < 40.0$	$40.0 \leqslant G < 60.0$	$G \geqslant 60.0$
Frequency	17	58	25

(i) By considering $\mathrm{P}(G < 40.0)$, write down an equation involving μ and σ. [2]

(ii) Find a second equation involving μ and σ. Hence calculate values for μ and σ. [4]

(iii) Explain why your answers are only estimates. [1]

4 A zoologist investigates the number of snakes found in a given region of land. The zoologist intends to use a Poisson distribution to model the number of snakes.

(i) One condition for a Poisson distribution to be valid is that snakes must occur at constant average rate. State another condition needed for a Poisson distribution to be valid. [1]

Assume now that the number of snakes found in 1 acre of a region can be modelled by the distribution Po(4).

(ii) Find the probability that, in 1 acre of the region, at least 6 snakes are found. [2]

(iii) Find the probability that, in 0.77 acres of the region, the number of snakes found is either 2 or 3. [4]

5 A continuous random variable X has probability density function

$$\mathrm{f}(x) = \begin{cases} \frac{1}{2}\pi \sin(\pi x) & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that this is a valid probability density function. [4]

(ii) Sketch the curve $y = \mathrm{f}(x)$ and write down the value of $\mathrm{E}(X)$. [3]

(iii) Find the value q such that $\mathrm{P}(X > q) = 0.75$. [3]

(iv) Write down an expression, including an integral, for $\mathrm{Var}(X)$. (Do not attempt to evaluate the integral.) [2]

(v) A student states that "X is more likely to occur when x is close to $\mathrm{E}(X)$." Give an improved version of this statement. [1]

6 In a city the proportion of inhabitants from ethnic group *Z* is known to be 0.4. A sample of 12 employees of a large company in this city is obtained and it is found that 2 of them are from ethnic group *Z*. A test is carried out, at the 5% significance level, of whether the proportion of employees in this company from ethnic group *Z* is less than in the city as a whole.

(i) State an assumption that must be made about the sample for a significance test to be valid. [1]

(ii) Describe briefly an appropriate way of obtaining the sample. [2]

(iii) Carry out the test. [7]

(iv) A manager believes that the company discriminates against ethnic group *Z*. Explain whether carrying out the test at the 10% significance level would be more supportive or less supportive of the manager's belief. [2]

7 An examination board is developing a new syllabus and wants to know if the question papers are the right length. A random sample of 50 candidates was given a pre-test on a dummy paper. The times, t minutes, taken by these candidates to complete the paper can be summarised by

$n = 50$,

$\sum t = 4050$,

$\sum t^2 = 329\,800$.

Assume that times are normally distributed.

(i) Estimate the proportion of candidates that could not complete the paper within 90 minutes. [6]

(ii) Test, at the 10% significance level, whether the mean time for all candidates to complete this paper is 80 minutes. Use a two-tail test. [7]

(iii) Explain whether the assumption that times are normally distributed is necessary in answering

(a) part (i),

(b) part (ii). [2]

8 The random variable W has the distribution $\mathrm{Po}(\lambda)$. A significance test is carried out of the null hypothesis H_0: $\lambda = 3.60$, against the alternative hypothesis H_1: $\lambda < 3.60$. The test is based on a single observation of W. The critical region is $W = 0$.

(i) Find the significance level of the test. [2]

(ii) It is known that, when $\lambda = \lambda_0$, the probability that the test results in a Type II error is 0.8. Find the value of λ_0. [4]

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